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Synoptic Variability of the Thermodynamical State of Rybinsk Reservoir Water Masses

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Abstract—Presented is a brief characteristic of structurally new grid-box mathematical model for calculation of characteristics of hydrological regime of reservoirs of any size and residence time, morphological class and hydroeconomic purpose, with time resolution of 24 h and depth increment of 1 m. Presented are the results of its verification, as well as the fragments of results of diagnostic calculation of changes in vertical distribution of temperature, mineralization, and discharges of katabatic, density, wind drift and compensated flows in separate segments of four reaches of the Rybinsk Reservoir in case of weather changing for all seasons of two years with extreme flow.

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INTRODUCTION

As a rule, considerable spatial inhomogeneity and variability are typical of water characteristics in medium and large reservoirs. The hydrodynamics of such reservoirs is determined by numerous factors with intensity of influence varying in time and space. The majority of these factors are directly or indirectly related to weather-forming synoptic processes in the area of the reservoir. The dependence of water masses structure and composition on synoptic conditions is most clearly manifested in large valley reservoirs with complex morphology. The Rybinsk Reservoir within the Volga-Kama cascade of dams is an example of such reservoir. Large water area and shoreline indentedness, slow water exchange, and feed by the flow of three large rivers, the Volga, Mologa, and Sheksna, predetermine very complex hydrological regime of the reservoir [1, 6, 8] whose many features require additional research despite the fact that the reservoir has existed for more than 70 years.

One of actual ways of such research is mathematical modeling of hydrological processes in the reservoir, most fully simulating its hydrological regime. By now, numerous mathematical models have been developed in world limnology; one-dimensional models with limited accounting for hydrological peculiarities of reservoirs prevail. Among them there is no model with closed annual cycle of water mass transformation processes occurring specifically in different year seasons. Stationary two-dimensional hydrodynamical lake models prevail in the number of variants; sometimes they are supplemented by ecological block where the reservoir is divided into shallow- and deep-water boxes, for example, the model of Lake Pleshcheevo [9]. Box limnological models are widespread abroad [10–14]. Nowadays, as referred to the Rybinsk Reservoir hydrology, a two-dimensional (on horizontal plane) stationary model of water circulation used and upgraded at Papanin Institute of Biology of Inland Waters of Russian Academy of Sciences (IBIW) is the most well-known [4].

DESCRIPTION OF UNIVERSAL MODEL

The hydrological model of a reservoir (HMR-MSU / GMV-MGU model) was developed at the Department of Land Hydrology of Moscow State University. The algorithm of the model takes into account tens of physical processes in the reservoir and on its interfaces with the environment. The model was created accounting for the following requirements:

1) schematization of the reservoir must represent its morphological structure and peculiarities of regulation of water intake from deep or surface (through a weir) reservoir layers;

2) simplicity of mathematical description of algorithm of numerical solution of equations;

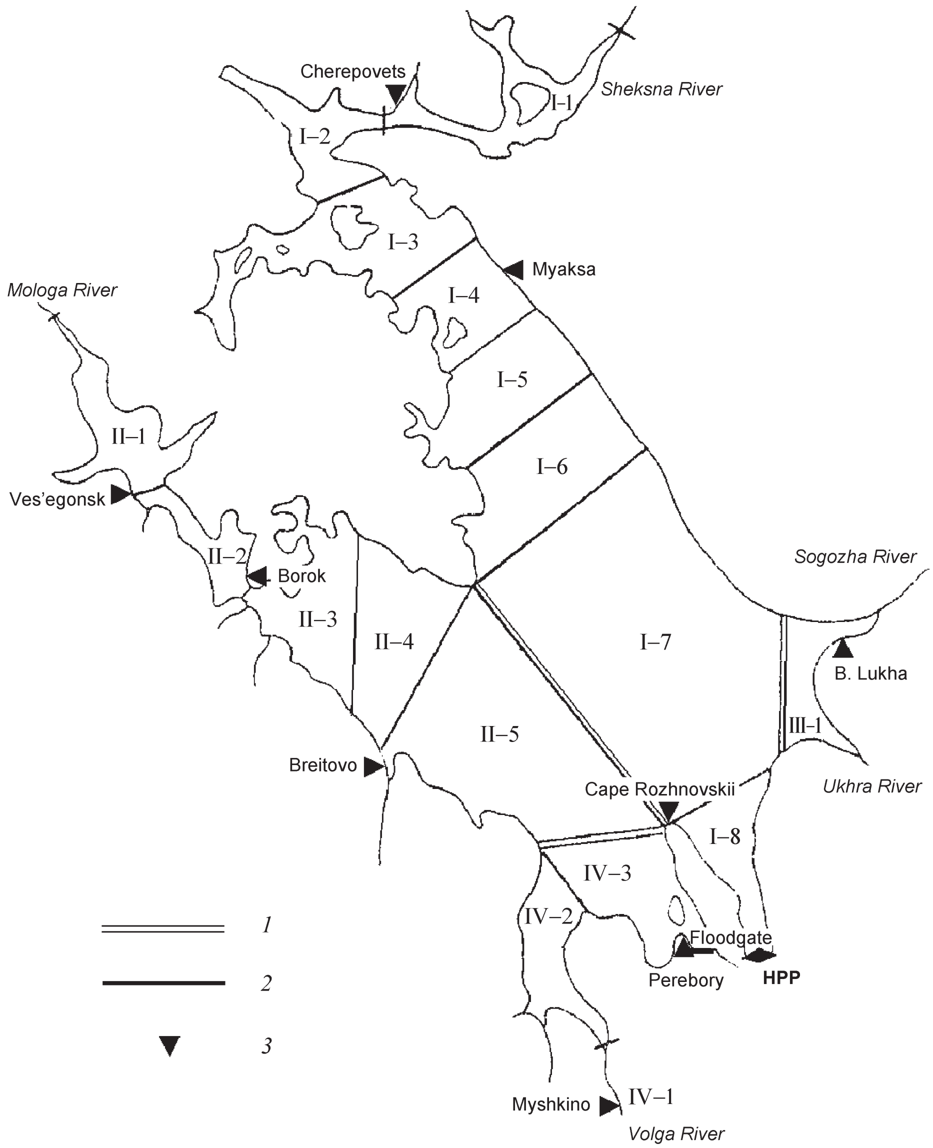

Fig. 1. Scheme of the Rybinsk Reservoir. Double lines are the boundaries of arms singled out (Roman numerals) from the data of [8]; single lines are the boundaries of model segments (Arabic numerals); triangles are hydrological gaging sites.

3) adequate imitation of main processes determining the hydrological regime of a reservoir (water stage fluctuations, fluctuations of the reservoir water balance components, peculiarities of internal water exchange, mineralization and water temperature regime, thermal balance, and ice phenomena);

4) simulation of reservoir water masses vertical structure with depth increment of 1 m and time resolution of 24 h;

5) description of hydrological processes must be based on the techniques recommended for hydroeconomic calculations, and model computations, on standard hydrometeorological information of Roshydromet.

Proceeding from these requirements, the reservoir is schematically represented either as a separate arm (a morphologically simple water body) or as a set of arms joining each other representing the inundated valleys of the river and its main tributaries in a morphologically complex reservoir. Each arm is divided lengthwise into segments (Fig. 1) accounting for morphometric peculiarities of the arm. Each segment is divided along the vertical into horizontal boxes; water mass is supposed to be homogeneous within each box.

The model is based on one-dimensional algorithm of computation of reservoir vertical structure [11]; this algorithm is applied consecutively to separated segments of the reservoir. Water stage is supposed to be horizontal; it is computed as the function of initial stage and daily variation of water volume in the reservoir. The conditions of water inflow to a segment are determined by the ratio between its density and vertical density stratification in the segment; the inflow zone depth, by the critical Richardson number enabling to estimate the limit of hydrodynamical stability on the zone boundaries. Water discharge to the tailwater pool of the closing dam is carried out through water intakes of different level. Water intake zone depth and outflow velocity field are determined by the discharge value and density stratification stability in this zone. The model includes the possibility of taking water for economical purposes from boxes of any segment of the reservoir.

Mathematical structure of the model is based on balance equations representing the continuity of water environment and the law of conservation of matter and energy in each box of the segment on condition of total mixing of inflowing water with the water in the design box during 24 h. Calculation of water and thermal balance of the reservoir is based on regulatory documents. Intrareservoir hydrodynamics is determined by external heat and water exchange (water inflow from the catchment, exchange with the atmosphere and bottom, and technogenic water intake), water impact on the water surface, and spatial density inhomogeneity of the water masses.

Horizontal water exchange between the segments is taken into account as katabatic, wind drift, density and compensated transport. Katabatic flow discharge is determined from the water balance of the segment. Wind circulation of the water in the reservoir is calculated using an algorithm of hydrodynamical block of IBIW software [4] included in the model. It is based on the stationary model of wind drift flows (full fluxes) adapted for the Rybinsk Reservoir. For numerical solution of the problem, the reservoir water area (except for seven segments in the upper reaches) is covered with a square grid with a step $\Delta x = \Delta y = 2$ km along x- and y-axis. Calculation is carried out for every 24 h. Absolute value of flow velocity vector and the angle between the vector and the x-axis are determined at each internal grid point; after that, discharges of water wind drift are calculated for each box of a segment on its boundary with adjacent segment (segments) of a reservoir.

Conditions favoring the formation of density flows in a valley reservoir remain practically through the whole year. The velocity of their distribution is calculated using the formula of nonuniform steady density flux. In the segments, both wind drift and density flows induce two lengthwise water circulations including upwellings, downwellings, and compensated counterflows closing them. Resultant water transport by these two counterflows is calculated in the model as an algebraic sum of water discharges participating in wind and gravitational circulations.

Vertical water exchange between the boxes in the segments is determined by nonstationarity of the processes of horizontal inflow and outflow of water [5], dynamical mixing in a katabatic flow, effective turbulent mixing, free convection, and induced convection in the form of Langmuir circulation. Dynamical mixing in a katabatic flow and effective turbulent mixing induced by different factors, mostly wind, influencing the water environment are estimated by finite-difference solution of equation of transport. Vertical water transport rates are given from the empirical relationships [5]. Free convective mixing in case of density instability is calculated from the balance scheme of full mixing of the layers involved into the convection. The depth of penetration of Langmuir circulation, which is one of key mechanisms of dynamics of reservoirs with long retention time, is determined using the technique presented in [7].

MODEL VERIFICATION

Hydrological regime of the Rybinsk Reservoir was calculated using the universal box grid model for two years with extremely high and extremely low flow (1962 and 1964, respectively). These calculations demonstrated that in 72% of cases, the error of reservoir daily water balance is less than the error of water accumulation estimation from the mean level change. Only 8% of cases demonstrated the exceedance of the accumulation estimation error corresponding to the error of determination of mean reservoir water level greater than ±2 cm. The majority of these cases took place during spring flood or in the periods of the fall floods, when the accuracy of estimation of mean reservoir water level is minimum.

Due to the fact that the Rybinsk Reservoir is a large one, the changes in ice depth, as well as in the depth of snow on ice, depend on the area of the reservoir. Snow depth variation range amounts to 30–40 cm according to the measurement data, and that of ice depth, to 55–65 cm. Comparison of snow and ice depth calculation results for main reaches of the reservoir with the streamgaging data demonstrated that calculation results are fairly satisfactory for the Main Reach. Standard error of ice and snow depth calculation S is equal to 4 cm, and the ratio S/σ between standard error S and standard deviation σ is equal to 0.27 for ice and 0.64 for snow. Considerably worse results are obtained for river reaches of the reservoir due to two reasons. First, the reaches are far from Rozhnovskii mys reference weather station whose data were used in the model calculations; second, the model does not take into account the effect of flow velocity on ice depth in narrow and shallow segments in upper reaches of reservoir arms. The differences in calculated and observed dates of ice appearance and disappearance at certain sites do not exceed 3 days in most cases.

Initial conditions and water temperature and electrical conductivity calculation results convergence with observational data were estimated from the results of hydrological surveys of the reservoir carried out by the employees of the IBIW, as well as from en-route and streamgage observations of Rybinsk Hydrometeorological Observatory. Hydrological surveys were performed for the grid of 47–66 stations evenly distributed across the main part of the reservoir. Eight surveys were carried out in 1964. Averaging was carried out for daytime observational data in surface layer (in the winter, for the data of individual days at separate stations located within one calculation segment). Maximum deviations of calculated water temperature values from observed ones were obtained for mid-May (up to 2.9°C) due to residual impact of ice drift in the reservoir, mainly from Molozhskaya arm (Mologa arm). For the summer and fall, the water temperature computation error had not exceeded 1.0–1.5°C and the observed temperature range was 5–21°C. The results of comparison of calculated and measured water mineralization values also indicate fairly satisfactory calculation quality: in 30% of 66 comparison cases, the error was not greater than 10 mg/l, in 50% of cases, not greater than 30 mg/l, and in 92% of cases, not greater than 50 mg/l.

Five surveys were carried out in 1962. From all surveys data, vertical distribution curves of box-averaged model water temperature and mineralization values for surveying day were plotted for each segment; the points denoting the values measured at all stations within a segment were also shown on the graphs. Given as an example in [3], the calculated and observational data from two surveys (May 12 and August 6) in the segment I–7 of the Main reach of the reservoir do not differ considerably from the similar comparison results for other segments and surveys. For several stations, the measured values coincide with calculated ones. Data from other stations located mostly at the periphery of segments differ from calculated values by 2–3°C and 30–50 mg/l due to different (and sometimes very small) "weights" of each station box-averaged values. Moreover, calculated values included all calculation errors of daily values of water temperature and mineralization for inflowing rivers, calculation errors of mineralization from electrical conductivity, as well as small water-balance model calculation errors. Accounting for this, as well as for slight (in the authors' opinion) discrepancies, one may state that calculated daily values of water temperature and mineralization in the Rybinsk Reservoir segment boxes are suitable for describing the synoptic variability of dynamics, structure, and composition of water masses of the Rybinsk Reservoir.

SYNOPTICAL VARIABILITY DIAGNOSIS

Synoptical variability of water masses is regular variation of their characteristics as a result of changing macroscale atmospheric processes, cyclones, anticyclones, and frontal zones. Its time scale range is from 24 hours to several weeks.

For the Rybinsk Reservoir, the year of 1962 was characterized by high water volumes not only due to springflood but also because of large water volumes of the summer and fall floods. This was also a cold year with maximum daily air temperature not greater than 20°C. During the ice-free period, half of the days were rainy and the precipitation amount exceeded 500 mm. Only six rainless periods that lasted more than 5 days were observed: three periods in May, one period in September, and two ones in November. Well pronounced phases of temperature rise and fall were observed only six times; air temperature fluctuations were insignificant during seven synoptical cycles. Ice-free period precipitation amount in the year of 1962 (low-flow year) was about 430 mm; maximum daily air temperature reached 25°C. There were also six long rainless periods (one period in May, one period in June, one period in August, one period in October, and two periods in July). Total cloud amount was 16% less than that on 1962 and lower cloud amount, 25% lower. Due to this fact, solar radiation coming to the water surface was 12% greater.

In the winter period, when the reservoir is covered with ice, the atmosphere does not practically affect the characteristics of water masses. Atmospheric influence can be recognized only in the spring, when snow disappears from the ice surface. According to the calculations, in 1962 snow disappearance took place in the beginning of the first ten-day period of April; in 1964 it occurred in the end of the first ten-day period of April, several days before the start of the spring filling of the reservoir. By the moment of ice cover breaking in the segments (the last ten-day period of April and the first ten-day period of May), their surface layer temperature reached 1.5–2.5°C due to sub-ice heating. Water temperature in upper reaches of the arms reached 3.5–4.0°C due to the inflow of warmer river waters of arm-forming tributaries. The temperature of bottom water layers in deep-water segments is of the same order of magnitude, whereas its values may be substantially lower in intermediate layers, forming peculiar vertical distribution of water temperature,

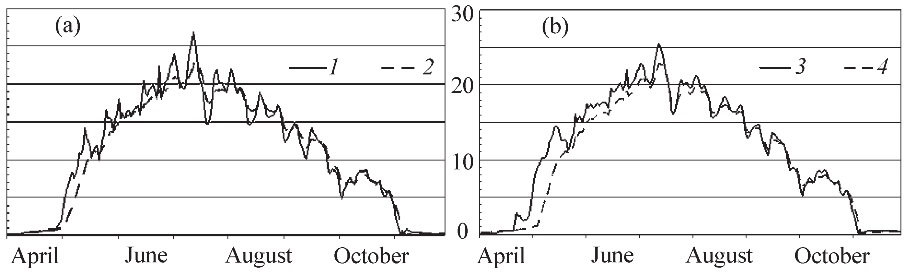

Fig. 2. Mean daily water temperature (°C) in the surface boxes of the Rybinsk reservoir model in 1964: (a) the segments (*1*) I–1 and (*2*) I–7 of the Sheksna arm; (b) the segments (*3*) II–1 and (*4*) II–5 of the Mologa arm.

dichotermy. In this case, density stability in the segments with such water temperature distribution due to large differences in the mineralization of surface and bottom layers, that had formed in the winter period and intensified due to the inflow of springflood melt waters to the surface layers of the reservoir. Vertical distribution of water temperature in the middle-reach segments of reservoir arms is described as direct stratification or isothermy; upper-reach segments are characterized by entirely homogeneous distribution of both water temperature and mineralization due to intensive dynamical mixing of water masses.

Downward solar radiation is the determining synoptical factor affecting the characteristics of reservoir water masses. The variability of this factor primarily depends on the features of cloudiness and temperature of air masses passing over the water area and the coast of the reservoir.

Sharply increasing synoptical influence on water masses of the reservoir is registered after the ice disappearance. During the first 10–15 days, intensive heat accumulation takes place; direct temperature stratification forms in the reservoir that is rapidly destroyed by drift and rapidly recovered after the wind impact weakening as well. Water temperature increase in the upper layers of the reservoir amounted to 0.46°C per day in 1962 (cyclone weather type) and in 1964, to 0.72°C per day in 1964 (anticyclone weather type). The depth of seasonal temperature jump layer in the segments of the reservoir is established during these two weeks, as a rule. The wind and air mass temperature over the reservoir have the most substantial influence on the characteristics of the water masses, because they determine the depth of wind mixing and the value of additional heat flux to the reservoir resulting from atmospheric turbulent exchange.

When water surface temperature is close to mean daily air temperature, the period of summer heating of the reservoir begins. Since this moment, atmospheric turbulent exchange becomes the negative component of heat balance and wind mixing becomes ordered convection. This leads to sharp slow-down of water column heating: it amounted to 0.13°C per day in 1962 and to 0.20°C per day in 1964. The periods of summer heating and summer and fall cooling are characterized by frequent alternation of reservoir heating and cooling phases of different intensity which are entirely determined by variability of weather conditions. Time scale of phases of synoptical cycles in 1962 and 1964 was from several days to two weeks. As moving downstream from upper reaches of reservoir arms to the Main reach, the amplitude of synoptical fluctuations of water temperature decreased (Fig. 2). Average intensity of summer and fall cooling of the reservoir amounted to 0.15°C per day in 1962 and to 0.21°C per day in 1964.

Synoptical variability of water temperature in the reservoir is more thoroughly described in [3] by the example of two adjacent phases of the same synoptical cycle: air temperature rise and fall from May 20 to June 9, 1962. As shown in Fig. 2, calculated values of water temperature T_0 in the surface boxes of the Main reach (segments I–7 and II–5 in Fig. 1) are compared with those in the river reaches near the outlets of the Sheksna (segment I-3), Mologa (segment II–3), and Volga (segment IV–2) rivers. The heating of water surface layer took place during the 10-day rainless warm phase (May 20–29): water temperature increased from 10 to 14°C in the Main reach and from 11 to 17°C in the river reaches. During the first days of the cold phase (cooling from 20 to 14°C, June 2–4), the values of T_0 in both groups of reaches became close; as cooling continued, the rate of water temperature decrease in the river reaches exceeded that in the Main reach. Such alternation of the days with homogeneity of water temperature T_0 across the whole reservoir and, on the contrary, 3–4°C difference in the river and lake-type reaches is a peculiarity of reservoirs of depression and valley type, which can be revealed only through simulating their hydrological regime.

Therefore, lengthwise inhomogeneity of water temperature in the reservoir during warming phases forms mainly as a result of more rapid heating of upper-reach segments, and the temperature alignment, as a result of more rapid cooling, when warming is followed by temporary cooling. On the contrary, lengthwise

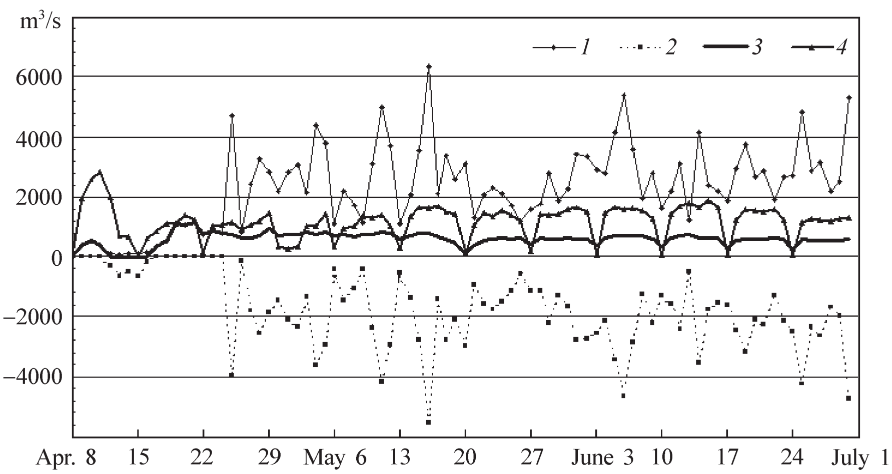

Fig. 3. Total daily water discharges (m^3/s) (*1*) of the flows towards the Rybinsk Dam and (*2*) counterflows upstream the reservoir through boundary cross section of the Sheksna arm between the segments I–6 and I–7, (*3*) their resultant, and (*4*) the Rybinsk Dam water discharges from April 8 to July 1, 1962.

inhomogeneity of water temperature during summer and fall cooling forms mainly as a result of more rapid cooling of upper-reach segments, and the temperature alignment, as a result of more rapid heating, when the cooling is interrupted by temporary warming. As the calculations demonstrated, the rainy year of 1962 was marked by temperature alignment multiple by 13 across the whole Rybinsk Reservoir water area during ice-free period, and low-flow warmer year of 1964, by that of multiple by 18.

Water mineralization variations during the synoptical cycle of weather change are related with genetic composition of river water masses and their mixing with waters of the Main reach. Water mineralization was increasing in the river reaches: from 123 to 144 mg/l in the Mologa reach (segment II–3) and from 78 to 97 mg/l in the Sheksna reach (segment I–3); it was slowly decreasing in the Main reach: from 167 to 162 mg/l in the segment I–7 and from 173 to 167 mg/l in the segment II–5. Judging by the calculated water discharges of katabatic, density, wind drift, and compensated flows across the boundaries of segments between the boxes, such variations of water mineralization are caused by water transport from one segment to another. One can estimate the fluctuations of sum total value of mean daily water discharges of flows of four types, directed downstream from the segment I–6 to the segment I–7 (Fig. 1) and vice versa, using Fig. 3. Resulting discharge, directed towards the segment I–7, is under the influence of sharp decrease of volumes of water released by the dam on weekends, which manifests itself in the scalloped form of the curve *3* (upward scallops mean the intensification of katabatic flows on business days of the week, downward scallops, decreasing flow). This influence can be traced on all the boundaries between calculation segments.

Weather influence on the changes of vertical structure of the main water mass of the reservoir in 1963 is shown by the example of two case studies: cyclone storm weather on May 13–17 (Figs. 4a–4c) with the velocity of eastern wind up to 12 m/s, and anticyclone sunny weather on August 4–8 (Figs. 4d–4f) with light wind of mainly southwest direction and velocity up to 3–4 m/s. In the first case, water temperature of the epilimnion decreased by 1.4°C, depth of thermocline increased by 4 m, and water density lapse rate $d\rho/dz$ increased twofold. Westerly wind drift flow circulation with the velocity up to 16 cm/s and easterly wind drift flow circulation with the velocity up to 11 cm/s formed in the Main reach. Westerly wind drift flows were spreading down to the depth of 3 m, and easterly ones, down to 10 m. Katabatic and compensated flows, according to the model algorithm, are spreading across the whole water column, since they are determined by the horizontal gradient of hydrostatic pressure, when the surface water slope forms. Compensated flow off windward western shore from the segment II–5 to the segment I–7 (Fig. 1) had the same direction as the katabatic flow; mean velocity of each flow amounted to 0.7–0.8 cm/s. In the second case, the changes of hydrological structure are less pronounced. Wind circulation was very weakly developed. Wind drift flow was spreading northeastwards and penetrating not deeper than 1 m; its mean velocity did not exceed 7 cm/s. The counterflow opposite in direction to the wind drift flow was formed at the depth of 3–7 m with maximum velocity of 1 cm/s at the depth of 4–5 m. The katabatic flow of 1 cm/s velocity was directed towards the dam, and the compensated flow of 0.5 cm/s velocity, off the dam. Slight increase in epilimnion temperature by 0.5°C took place.

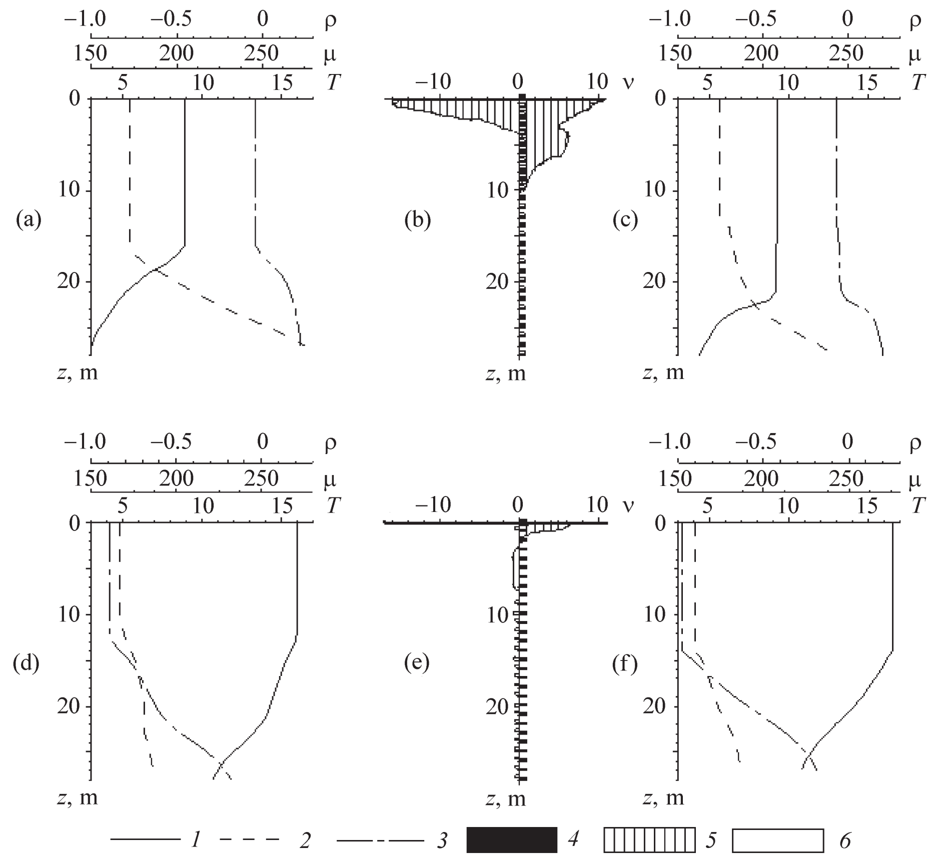

Fig. 4. Vertical distribution of (*1*) water temperature T, °C, (*2*) water mineralization μ, mg/l, (*3*) conditional water density ρ kg/m^3, and velocities (cm/s) of (*4*) katabatic, (*5*) wind drift and (*6*) compensated flows on (a) May 13, (b) May 15, (c) May 16, (d) August 4, (e) August 6, and (f) August 8, 1962. Positive values of flow velocity correspond to the easterly flows, and negative ones, to the westerly ones.

CONCLUSIONS

The analysis of performed diagnostic calculations shows that synoptical variability influence on the characteristics of water masses in different parts of the reservoir starts since the moment of snow disappearance from the ice cover. After the ice disappearance, synoptical variability practically entirely determines the fluctuations of intensity of water masses heating or cooling, the formation and stability of vertical density stratification, the depth and dynamics of seasonal and synoptic layers of temperature jump, the depth of homogeneous epilimnion, as well as the structure and intensity of wind drift and compensated flows. Synoptical variability has indirect influence (through affecting the water masses of the main tributaries) on the inflow conditions, hydrodynamics, and chemical and temperature regime of the upper reaches of the reservoir arms.

The use of GMV-MGU model, which combines the possibility of simulating the lengthwise variations of water characteristics and the routine estimation of wind drift and compensated flow structure, enables to carry out the first quantitative estimation of the weather conditions influence on the characteristics of water masses in any part of the reservoir with a fairly small time step, 24 hours. This is of special importance for performing diagnostic and prognostic calculations of hydrological regime of present or designed large reservoirs of depression and valley type with complex morphology under various schedules of changes in their operating rules, taking into account the scenarios of climate change [2].

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